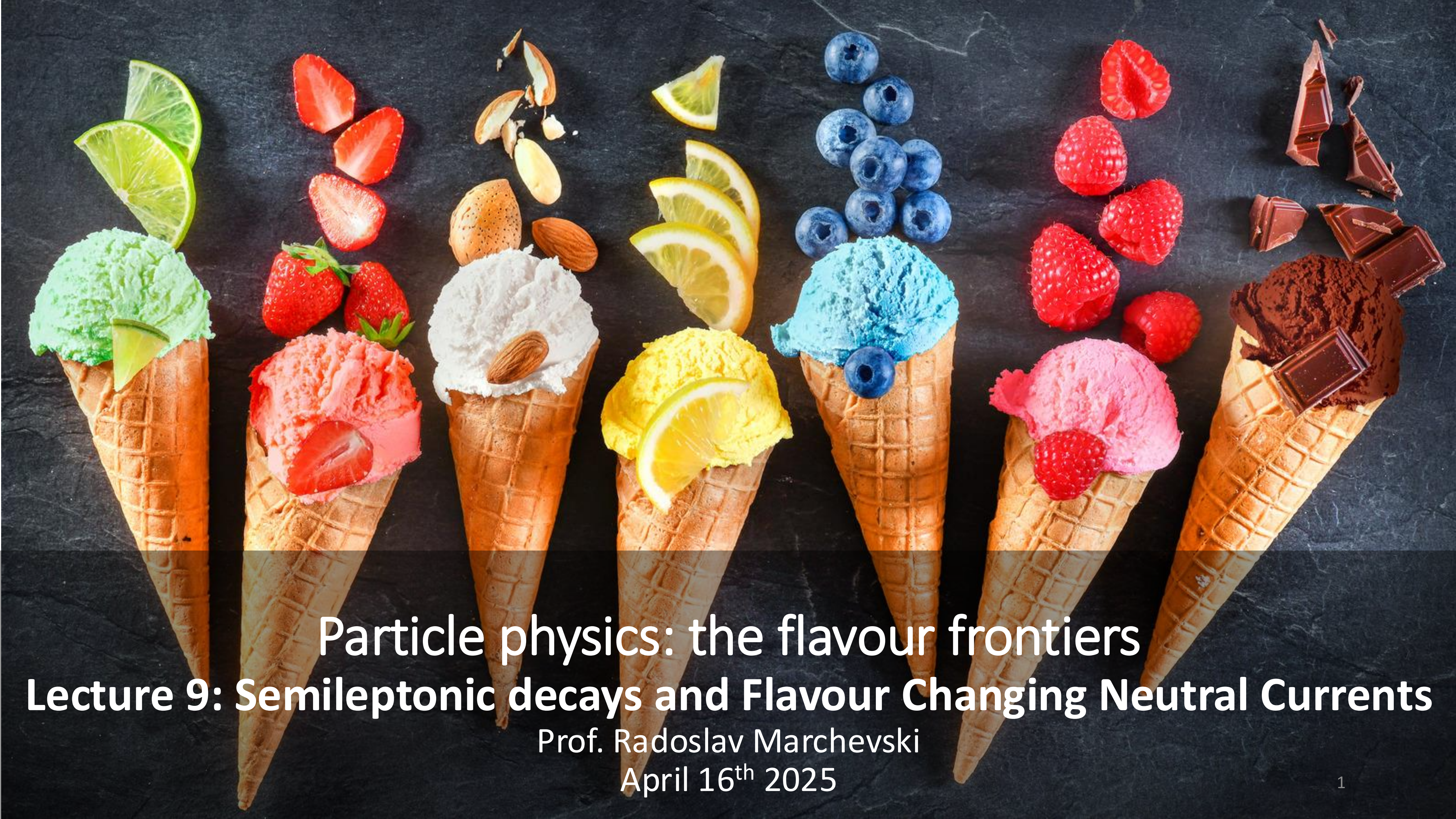


Particle physics: the flavour frontiers

Lecture 9: Semileptonic decays and Flavour Changing Neutral Currents

Prof. Radoslav Marchevski

April 16th 2025

Short recap and today's learning targets

Last time we discussed

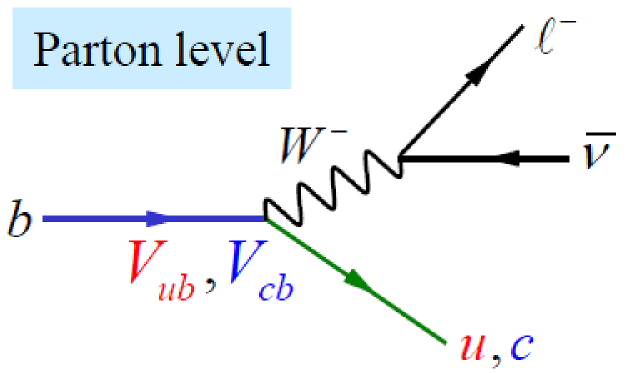
- What are some of the main characteristics of QCD at low energy
- Interplay between QCD and weak interactions in meson decays (factorization, decay constants, FFs)
- Leptonic decays of charged mesons and which experimental techniques can be used to measure them

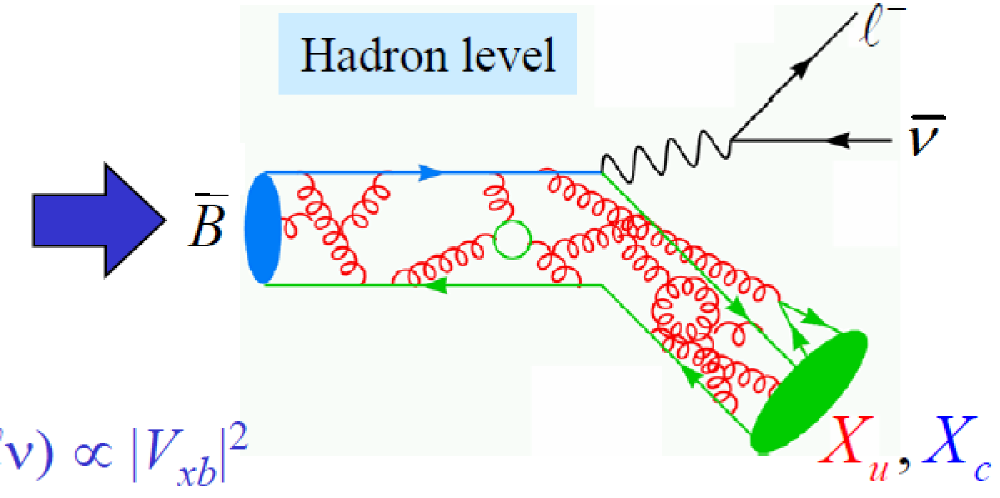
Today you will …

- learn about tree-level semileptonic decays of mesons, how to measure their properties experimentally, CKM tests and observed tensions in the data
- get familiar with Flavour Changing Neutral Currents and how do they appear in the Standard Model

Semileptonic (tree-level) meson decays

$K \to \pi l \nu, \qquad D \to X_{s,d} l \nu, \qquad B \to X_{u,c} l \nu$





■ Decay rate $\Gamma_x \equiv \Gamma(b \to x \ell \nu) \propto |V_{xb}|^2$

Different theoretical and experimental approaches depending on the flavour

Form factors

- Encode the non-perturbative part of the hadronic matrix element (can be calculated by lattice QCD)
- We can use approximate symmetries of QCD to learn more about them and relate them to each other
- The physics intuition is that form factors arise from the overlap of the wave function of the two hadrons
  - from QM: probability of a fast transition between two states $i \to f$ depends on the overlap between their wavefunctions
- The sudden transition in semileptonic hadron decays is due to the weak interaction

Semileptonic kaon decays

- **Decay rate:** $\Gamma(K \to \pi l \nu) = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW}(1 + \delta_K^l + \delta_{SU2}) C^2 |V_{us}|^2 f_+^2(0) I_K^l$

$C = 1(K^0), 1/2(K^+)$

S_{EW}, δ_K^l: SD, LD electromagnetic corrections

δ_{SU2}: Isospin breaking correction

$f_+^2(0)$: Form factor at zero momentum transfer

I_K^l: Phase space integral

dynamics described by 2 form factors (FF)*

$$f_+(t) = f_+(0)\left(1 + \lambda'_+ \frac{t}{m_{\pi^+}^2} + \lambda''_+ \frac{t^2}{m_{\pi^+}^4}\right), \qquad f_0(t) = f_+(0)\left(1 + \lambda_0 \frac{t}{m_{\pi^0}^2}\right)$$

t: 4- momentum transfer $K - \pi$

λ'_+, λ''_+, λ_0: Form factor parameters (enter I_K^l)

- **Experimental strategy** to extract V_{us} ($\phi -$ factory and fixed target)
  - Selection of $K \to \pi l \nu$ decays: background $<$ %, acceptance well-reproduced by simulations
  - Measurement of branching ratio: normalising to the luminosity ($\phi -$ factory) or another decay (fixed target)
  - Measurement of the FF parameters $\lambda_{+,0}$: fit the (E_l^*, E_π^*) Dalitz plot density
  - Theoretical inputs: $S_{EW}, \delta_K^l, \delta_{SU2}, f_+^2(0)$ (low energy EFT, lattice QCD calculations)

* Here Taylor parametrization of the FFs; different types of parametrization exist in the literature

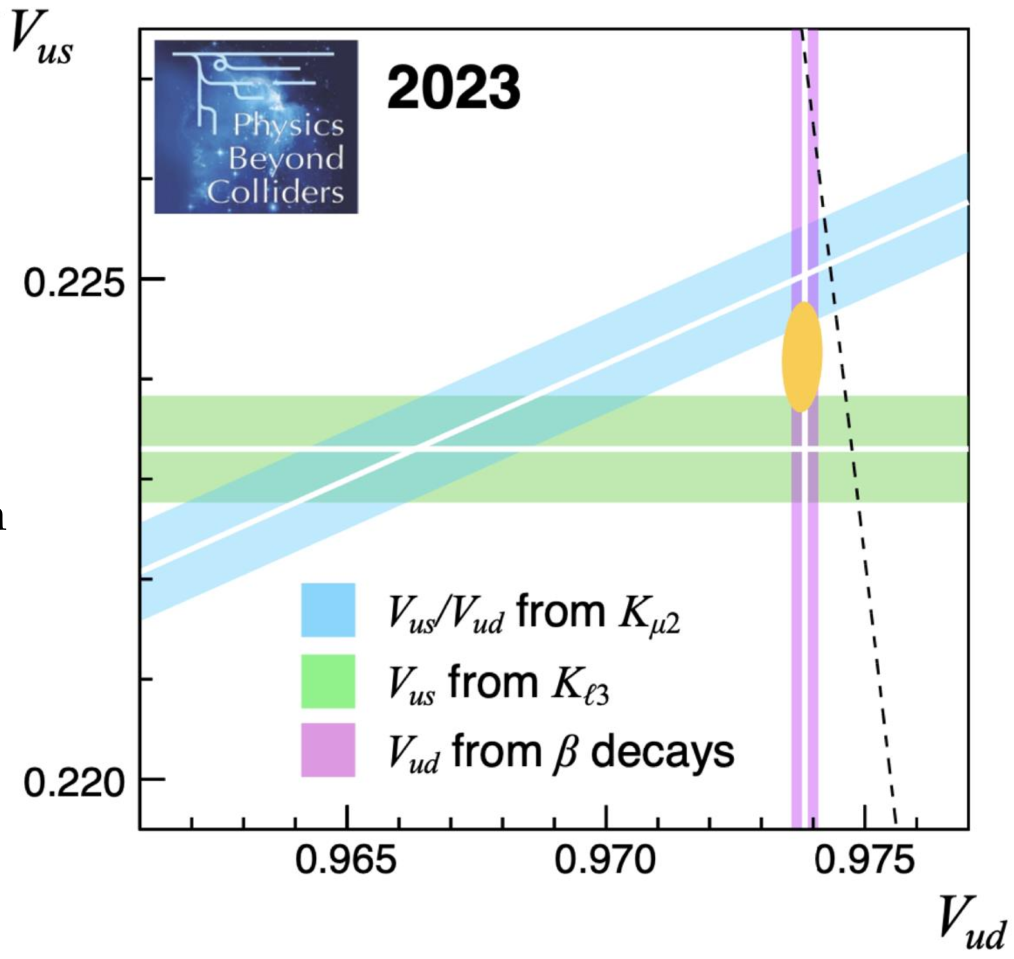
Semileptonic kaon decays

- **Standard acronyms:** $\Gamma(K \to \pi l \nu) \to K_{l3}$, $\Gamma(K \to l\nu) \to K_{l2}$, $\Gamma(\pi \to l\nu) \to \pi_{l2}$
- $|V_{ud}| = 0.97373(31)$ from superallowed β transitions $(0^+ \to 0^+)$
- First row CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)(4)$$

$\approx 1.5 \times 10^{-5}$

$\sim 2\sigma$ away from unitarity



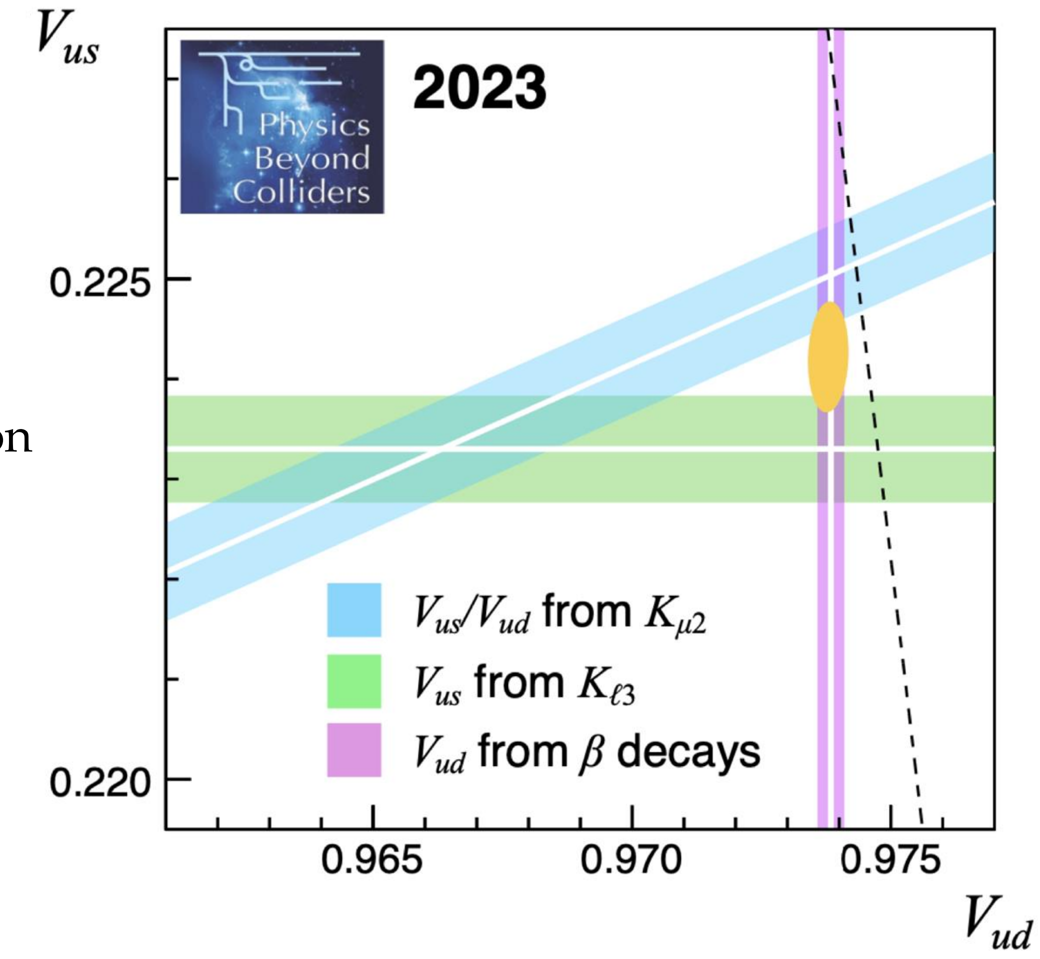
- We can ignore $|V_{ub}|$ to the level of achievable experimental precision
- Small deviation (two standard deviations) from unitarity observed
- What could the possible problems leading to this discrepancy be?

Semileptonic kaon decays

- **Standard acronyms:** $\Gamma(K \to \pi l \nu) \to K_{l3}, \quad \Gamma(K \to l\nu) \to K_{l2}, \quad \Gamma(\pi \to l\nu) \to \pi_{l2}$
- $|V_{ud}| = 0.97373(31)$ from superallowed β transitions $(0^+ \to 0^+)$
- First row CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)(4)$$

$\approx 1.5 \times 10^{-5}$ ~2σ away from unitarity

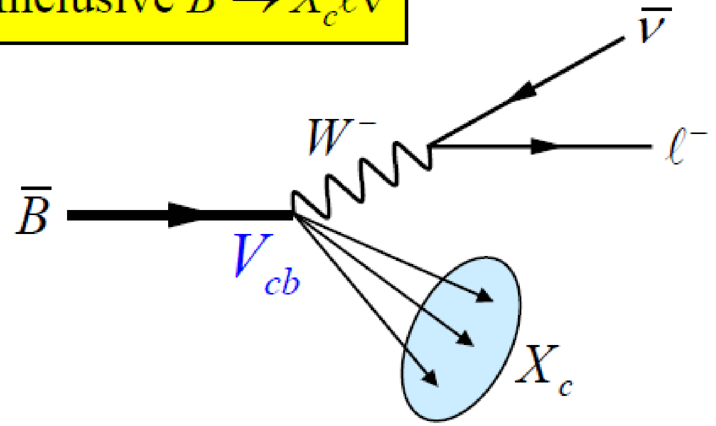


- We can ignore $|V_{ub}|$ to the level of achievable experimental precision
- Small deviation (two standard deviations) from unitarity observed
- What could the possible problems leading to this discrepancy be?
  - Statistical fluctuation?
  - Problem in experimental inputs?
  - Problem in theoretical inputs?
  - New physics?

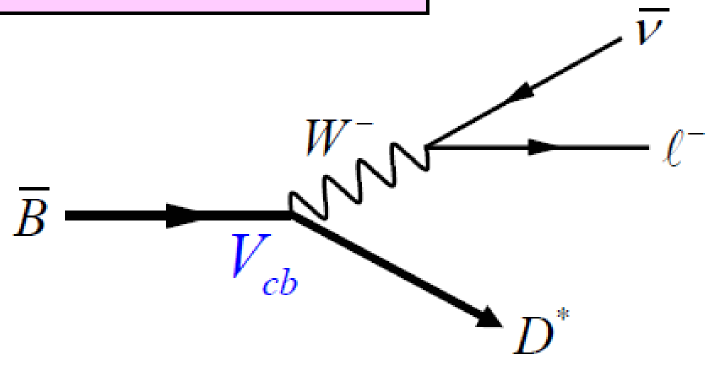
Semileptonic $\boldsymbol{B}$ meson decays

Inclusive vs exclusive: two different theoretical and experimental approaches

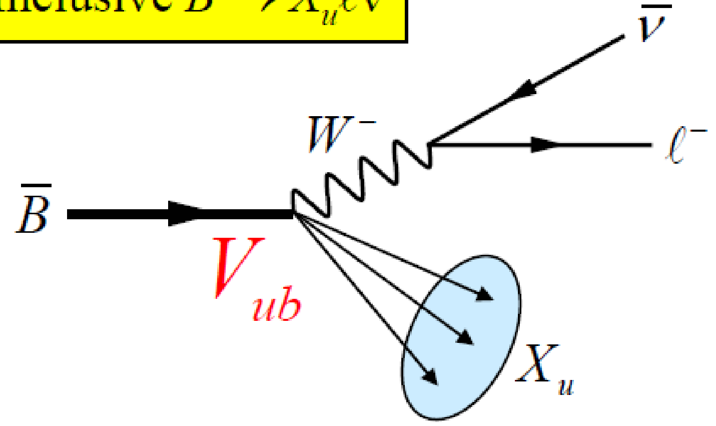
Inclusive $B \to X_c \ell \nu$



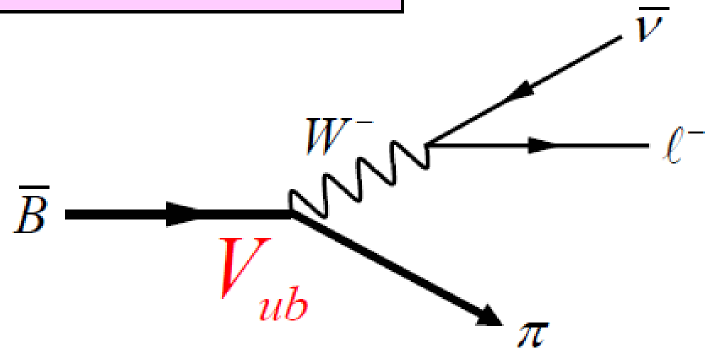
Exclusive $B \to D^* \ell \nu$



Inclusive $B \to X_u \ell \nu$



Exclusive $B \to \pi \ell \nu$



Semileptonic $\boldsymbol{B}$ meson decays to charm: V_{cb} inclusive

EM corrections

QCD perturbative correction

- **Inclusive** (OPE*)

$$\Gamma = |V_{cb}|^2 \frac{G_F^2 m_b^5}{192\pi^3} |\eta_{EW}|^2 \left[1 + \mathcal{O}\left(\frac{\alpha_s}{\pi}\right) + \mathcal{O}\left(\frac{1}{m_b^2}\right)\right]$$

non-perturbative expansion: depends on several parameters

- **Experimental strategy** to extract $|V_{cb}|$ (B factories)
  - *Challenge:* distinguish leptons from B or cascade (charmed mesons produced by B decays)
  - *Selection:* reasonably clean only in some part of the phase space
    - partial decay rates more sensitive to non-perturbative parameters
  - *Parameter determination:* analysis of the shape of the fit of the E_l and M_X distributions (lepton energy and hadron mass)
    - the "moments" of E_l and M_X distributions ($\langle E_l^n\rangle, \langle M_X^n\rangle$) depend on V_{cb}, m_b and the parameters
    - fit the E_l and M_X distributions and their moments to extract $\boldsymbol{V_{cb}, m_b}$ and the **parameters**
  - *Uncertainties:* parametrization, non-perturbative expansion

$*$ Operator Product Expansion

Semileptonic $\boldsymbol{B}$ meson decays to charm: V_{cb} inclusive

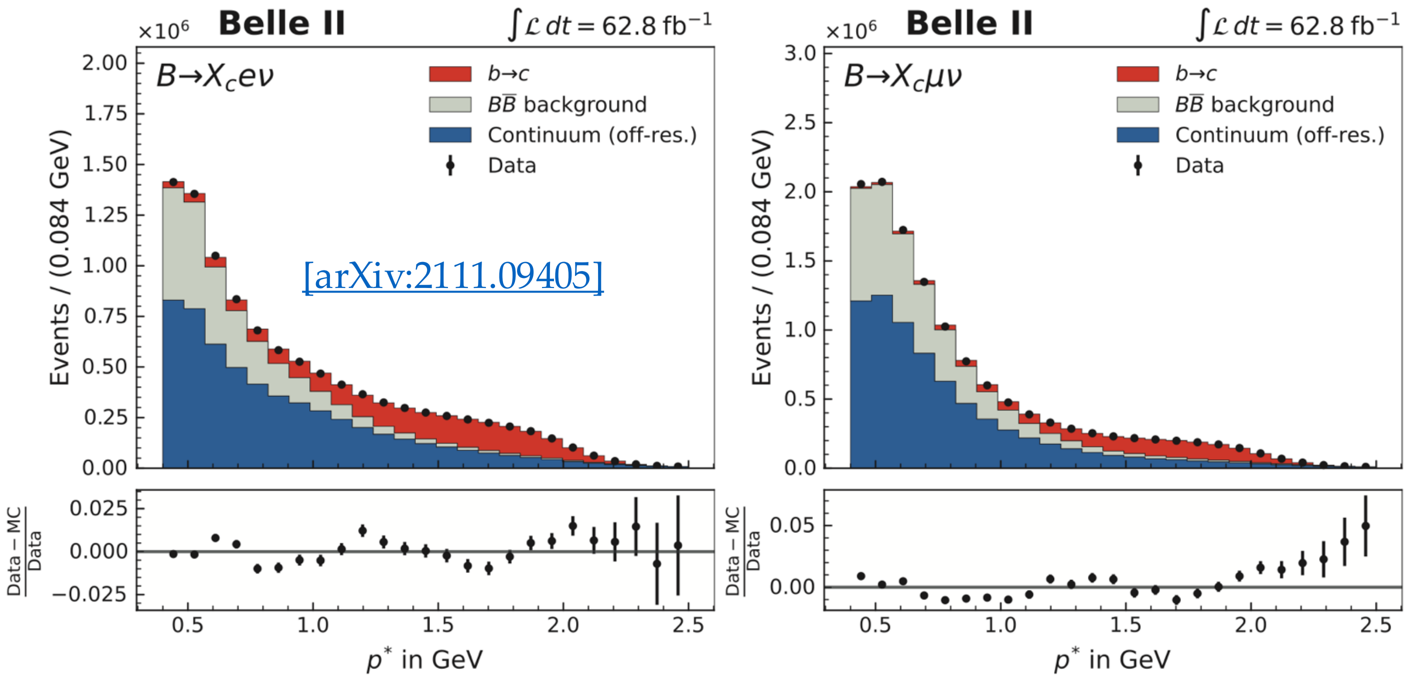
- **Inclusive**
  (OPE*)

$$\Gamma = |V_{cb}|^2 \frac{G_F^2 m_b^5}{192\pi^3} |\eta_{EW}|^2 \left[1 + \mathcal{O}\left(\frac{\alpha_s}{\pi}\right) + \mathcal{O}\left(\frac{1}{m_b^2}\right)\right]$$

EM corrections

QCD perturbative correction

non-perturbative expansion:
depends on several parameters



[arXiv:2111.09405]

Combination of all inclusive measurements

- $|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$ (inclusive)

$*$ Operator Product Expansion

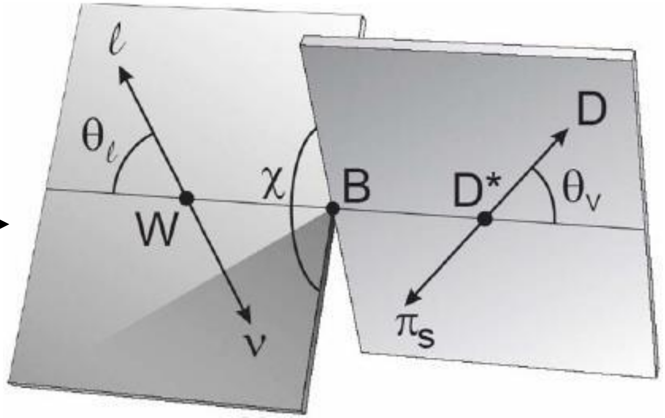
Semileptonic $\boldsymbol{B}$ meson decays to charm: V_{cb} exclusive

- **Exclusive** $\bar{B} \to D^* l \bar{\nu}_l$, $\bar{B} \to D l \bar{\nu}_l$, $\bar{B}_s \to D_s^{(*)-} l^+ \bar{\nu}_l$ → **LHCb ($\boldsymbol{l = \mu}$)**

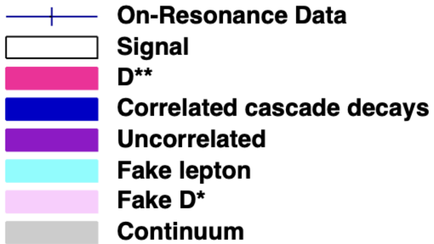
$$\frac{\mathrm{d}\Gamma}{\mathrm{d}w} \approx |V_{cb}|^2 \frac{G_F^2 m_b^5}{48\pi^3} |\eta_{EW}|^2 \mathcal{G}(w) |\mathcal{F}(w)|^2$$

- w: D boost in B rest frame
- η_{EW}: electromagnetic corrections (SD+LD) [from theory]
- $\mathcal{G}(w)$: phase space factor
- $\mathcal{F}(w)$: decay amplitude, depends on form factors (3 parameters)
  [$\mathcal{F}(1) \sim \mathcal{O}(1)$ from lattice QCD]

- **Experimental strategy** (example $\bar{B} \to D^* l \bar{\nu}_l$, B factories):
  - *Selection:* low background
  - *Relevant variables:* w, decay angles θ_l, θ_V, χ →
  - *Implementation:* simultaneous fit to the 4 variables to extract $|V_{cb}|$ and form factor parameters
  - *Uncertainties:* detection efficiency and D^* branching ratios, parametrisation of the form factors



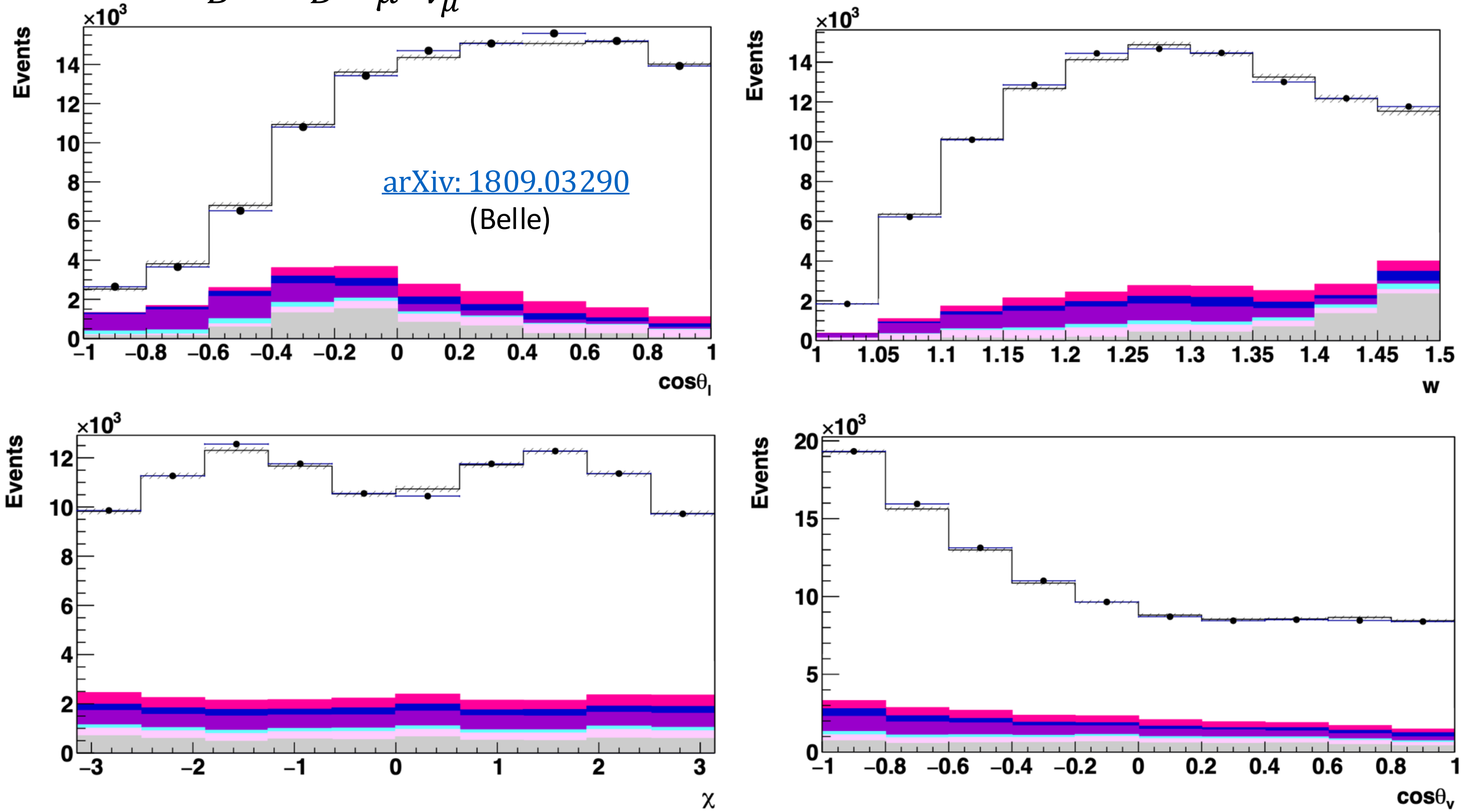
Semileptonic B meson decays to charm: V_{cb} exclusive



Combination of (most) exclusive measurements

- $|V_{cb}| = (39.4 \pm 0.8) \times 10^{-3}$ (exclusive)

$\bar{B}^0 \to D^{*-}\mu^+\bar{\nu}_\mu$

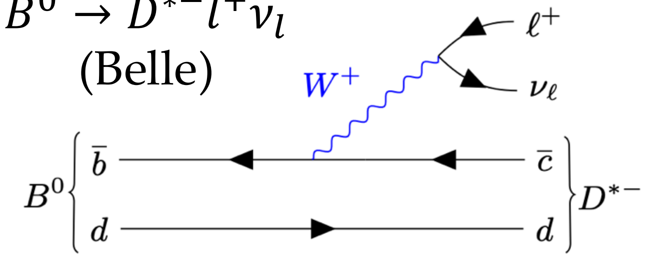


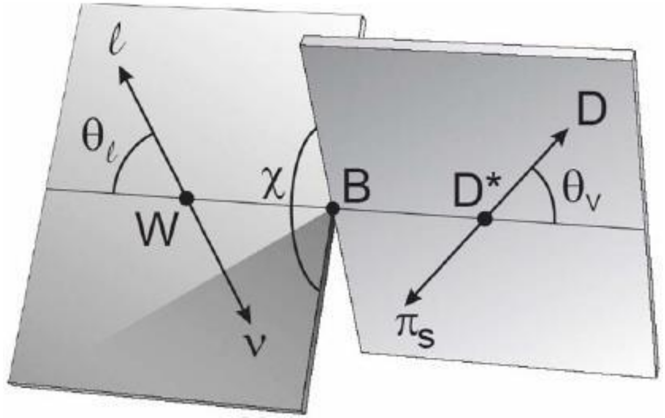
arXiv: 1809.03290

(Belle)

$\bar{B}^0 \to D^{*-}l^+\bar{\nu}_l$

(Belle)

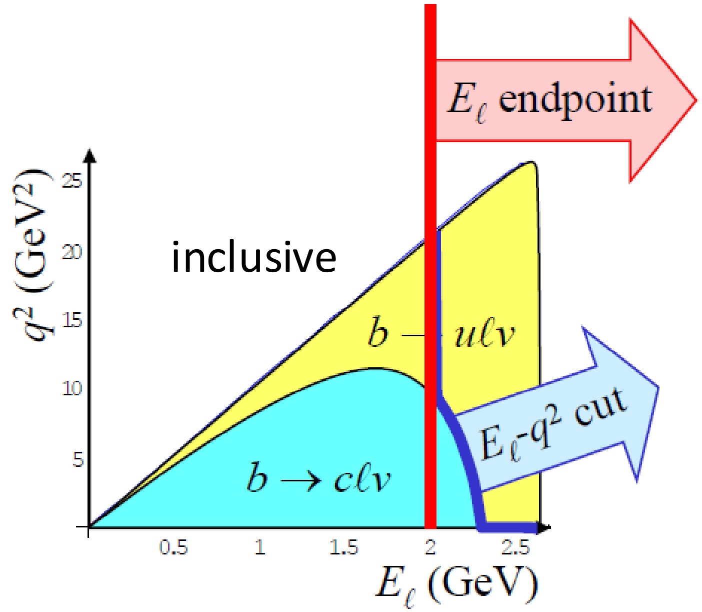




Semileptonic B meson decays to charm: V_{ub}

- $|V_{ub}|$ provides a measurement of $A\lambda^3\sqrt{\eta^2+\rho^2}$ in the Wolfenstein parametrisation
  - constraints in the (ρ, η) plane using tree-level measurements

- $B \to X_u l^+ \nu$: inclusive and exclusive approach similar to V_{cb} (B factories)
  - *Selection:* low statistics, large background from $B \to X_c l^+ \nu$ requires a selection of **small portions** of the phase space
  - partial decay rates are known with much larger theoretical uncertainty than total ones



$$|V_{ub}| = (4.13 \pm 0.14 \pm 0.18) \times 10^{-3} \text{ (inclusive)}$$
$$|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3} \text{ (exclusive)}$$

Theoretical uncertainty dominates

- Alternative measurements available $B \to \tau\nu$, $B \to K^-\mu^+\nu_\mu$, Λ_b^0 semileptonic decays

Semileptonic $\boldsymbol{B}$ meson decays to charm: V_{cb} summary

$|V_{cb}|$ provides a measurement of $A\lambda^2$ in the Wolfenstein parametrisation

- $|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$ (inclusive)
- $|V_{cb}| = (39.4 \pm 0.8) \times 10^{-3}$ (exclusive)

$|V_{cb}| = (40.8 \pm 1.4) \times 10^{-3}$ (average)

$|V_{ub}|$ provides a measurement of $A\lambda^3\sqrt{\eta^2 + \rho^2}$ in the Wolfenstein parametrisation

- $|V_{ub}| = (4.13 \pm 0.14 \pm 0.18) \times 10^{-3}$ (inclusive)
- $|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$ (exclusive)

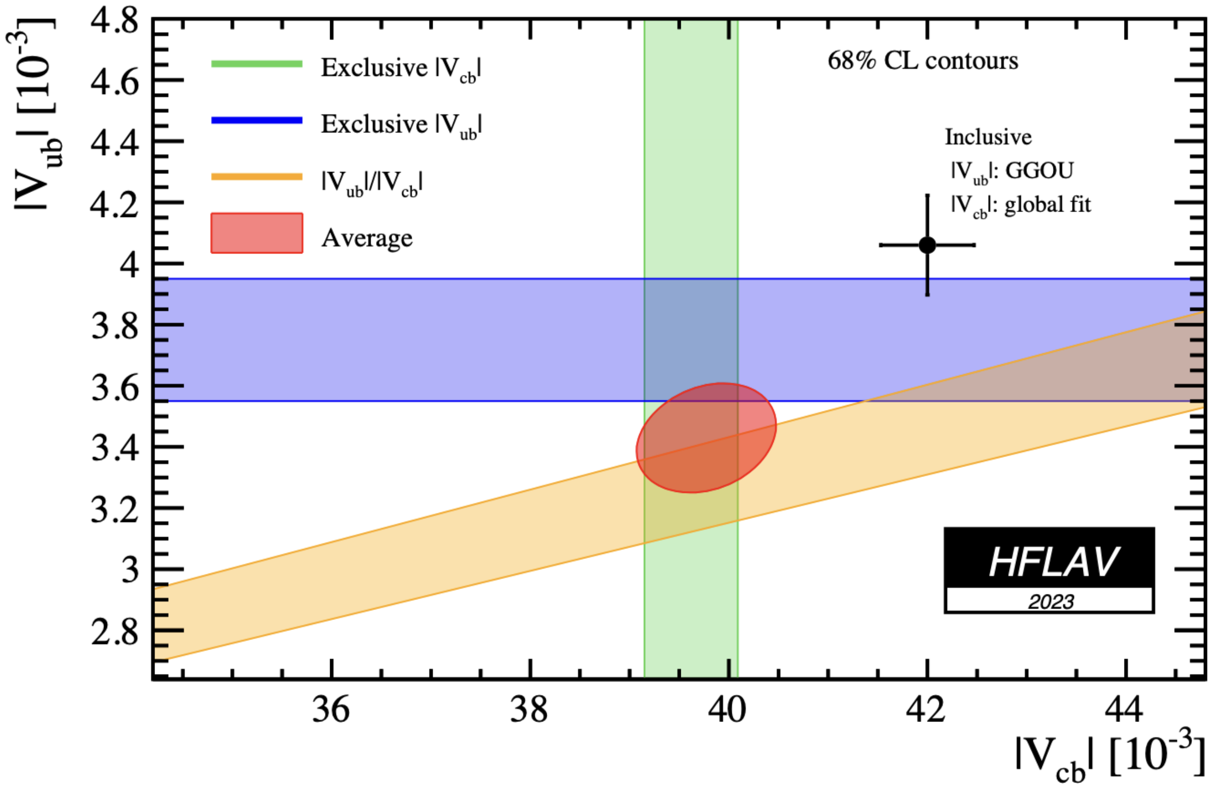
$|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ (average)

Semileptonic B meson decays to charm: V_{cb}, V_{ub} summary

- $|V_{cb}|, |V_{ub}|$ from previously discussed measurements

> 3σ discrepancy between exclusive and inclusive determinations

- What could the possible problems leading to this discrepancy be?
  - Statistical fluctuation?
  - Problem in experimental inputs?
  - Problem in theoretical inputs?
  - New physics?



- Long-standing puzzle: new measurements ongoing (LHCb, Belle II)

Flavour Changing Neutral Currents (FCNCs)

- Historically, the strong suppression of FCNCs played a very important role in the constructing the SM
- Now they continue to play an essential role in testing the SM and searching for New Physics (NP)

- **No FCNCs are allowed at tree level**
  - W-boson is charged and only couples to up-down (neutrino-charged lepton) pairs
  - only neutral bosons can mediate FCNCs
  - *four neutral bosons in the SM:* the photon, the gluon, the Z-boson and the Higgs-boson

- In the SM there is no symmetry that forbids FCNCs in the quark sector → loop contributions to these processes

- The photon and gluon have flavour diagonal and universal couplings
  - guaranteed by gauge invariance
  - the invariance of the kinetic terms requires universality of the gauge couplings related to the unbroken symmetries

Flavour Changing Neutral Currents (FCNCs): Z-boson

- The Z-boson corresponds to a broken symmetry $\rightarrow$ no fundamental symmetry that forbids Z-boson
  - however, we explicitly showed that in the SM the Z couplings are universal and diagonal. **How come?**

- Z couplings are proportional to $T_3 - Q\sin^2\theta_W$
- The mass eigenstates are characterised by spin, $SU(3)_C$ representation and $U(1)_{\mathrm{EM}}$ charge

- Q must be the same for all flavours in a given sector but there are two options for T_3
  - *Option 1:* all mass eigenstates in that sector originate from interaction eigenstates of the same $SU(2)_L \times U(1)_Y$ representation and have the same T_3 and Y
  - *Option 2:* the mass eigenstates in this sector mix interaction eigenstates with the same $Q = T_3 + Y$ but different $SU(2)_L \times U(1)_Y$ representations and in particular, different T_3 and Y

Flavour Changing Neutral Currents (FCNCs): Z-boson

- *Option 1:* all mass eigenstates in that sector originate from interaction eigenstates of the same $SU(2)_L \times U(1)_Y$ representation and have the same T_3 and Y

- Z couplings in the fermion interaction basis are universal $\Longrightarrow$ proportional to the unit matrix (times $T_3 - Q \sin^2 \theta_W$ of the relevant interaction eigenstates)

- The rotation to the mass basis maintains the universality

$$V_{fM} \times \mathbf{1} \times V_{fM}^{\dagger} = \mathbf{1}, \qquad (f = u, d; M = L, R)$$

Flavour Changing Neutral Currents (FCNCs): Z-boson

- *Option 2:* the mass eigenstates in this sector mix interaction eigenstates with the same $Q = T_3 + Y$ but different $SU(2)_L \times U(1)_Y$ representations and in particular, different T_3 and Y

- Z couplings in the fermion interaction basis are diagonal but not universal $\Longrightarrow$ each diagonal entry is proportional to the relevant $T_3 - Q \sin^2\theta_W$ factor

- Generally in this case the rotation to the mass basis does not maintain the diagonality

$$V_{fM} \times \widehat{\boldsymbol{G}}_{\mathbf{diagonal}} \times V_{fM}^{\dagger} = \widehat{\boldsymbol{G}}_{\mathbf{non-diagonal}}, \qquad (f = u, d; M = L, R)$$

Flavour Changing Neutral Currents (FCNCs): Z-boson

- *Option 1 (the Standard Model):* all mass eigenstates in that sector originate from interaction eigenstates of the same $SU(2)_L \times U(1)_Y$ representation and have the same T_3 and Y

- All fermions mass eigenstates with a given chirality and in a given $SU(3)_C \times U(1)_{\text{EM}}$ representation come from the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ representation

- *Example:* all left-handed up quark mass eigenstates, which are in the $(3)_{+2/3}$ come from interaction eigenstates in the $(3,2)_{+1/6}$
  - this is the reason that the SM predicts universal couplings to fermions
  - if there existed in Nature also left-handed quarks in the $(3,1)_{+2/3}$ representation $\Longrightarrow$ Z couplings in the left-handed up sector would be non-universal and the Z-boson could mediate FCNCs (e.g. $t \to cZ$) at tree level

Flavour Changing Neutral Currents (FCNCs): Higgs boson

- The Yukawa couplings of the Higgs boson are not universal (fully general 3×3 matrices in the interaction basis)
- In the fermion mass basis, the Higgs boson couplings are diagonal (the mass and Yukawa matrices are simultaneously diagonalized)!
- Condition for the absence of Higgs—mediated FCNCs: **single Higgs field**

- Features of the Standard Model
  - all SM fermions are chiral and therefore there are no bare mass terms
  - the scalar sector only has a single Higgs doublet

- Extensions that generate flavour-changing Higgs couplings
  - there are quarks and/or leptons in a vector-like representations and therefore bare mass terms are allowed
  - there is more than one $SU(2)_L$-doublet scalar that couples to a specific type of fermions

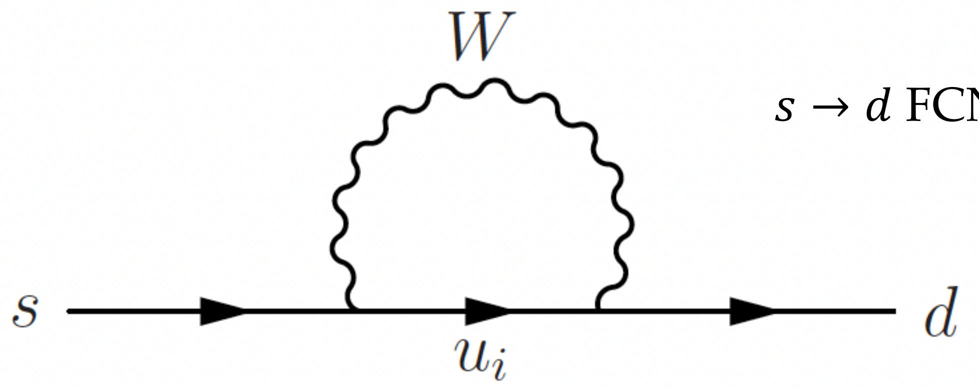
In the Standard Model all FCNC processes are loop suppressed

(in SM extensions FCNCs can appear at tree level, mediated by the Z-boson, Higgs boson, or new massive bosons)

Flavour Changing Neutral Currents (FCNCs) in the SM

- In the SM there is no symmetry that forbids FCNCs in the quark sector → loop contributions to these processes
  - W-mediated interactions at one-loop level
  - the W-boson couplings are charged current flavour changing an even number of insertions of W- boson couplings are needed to generate an FCNC process
  - no tree-level contributions → suppressed in the SM → large indirect sensitivity to New Physics

- FCNCs probe physics at scales much higher than the energy scale of the relevant experiments
  - feature which was demonstrated several times in the history of particle physics



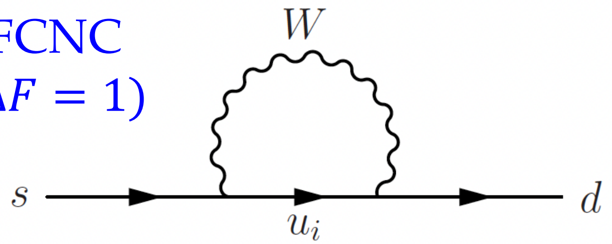
$s \to d$ FCNC transition

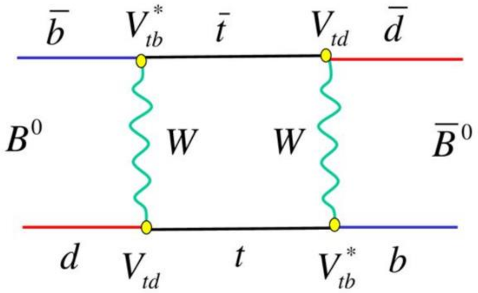
Flavour Changing Neutral Currents (FCNCs) in the SM

- FCNC processes played an important role in predicting the existence of SM particles before they were directly discovered, and in predicting their masses
  - the smallness of $\Gamma(K_L \to \mu^+\mu^-)$ led to predicting the charm quark
  - the size of the mass difference in the neutral kaon system Δm_K, led to a successful prediction of the charm mass
  - the measurement of CP violation in the kaon system led to predicting the existence of third generation fermions
  - the size of the mass difference in the neutral B system, Δm_B, led to a successful prediction of the top mass

- We will consider two classes of FCNCs based on the change in F (charge under the global $[U(1)]^6$ flavour symmetry of $\mathcal{L}_{\text{QCD}}$)
  - FCNC decays ($\Delta F = 1$ processes) have two insertions of W- boson couplings
  - Neutral meson mixings ($\Delta F = 2$ processes) have four insertions of W- boson couplings

FCNC
($\Delta F = 1$)

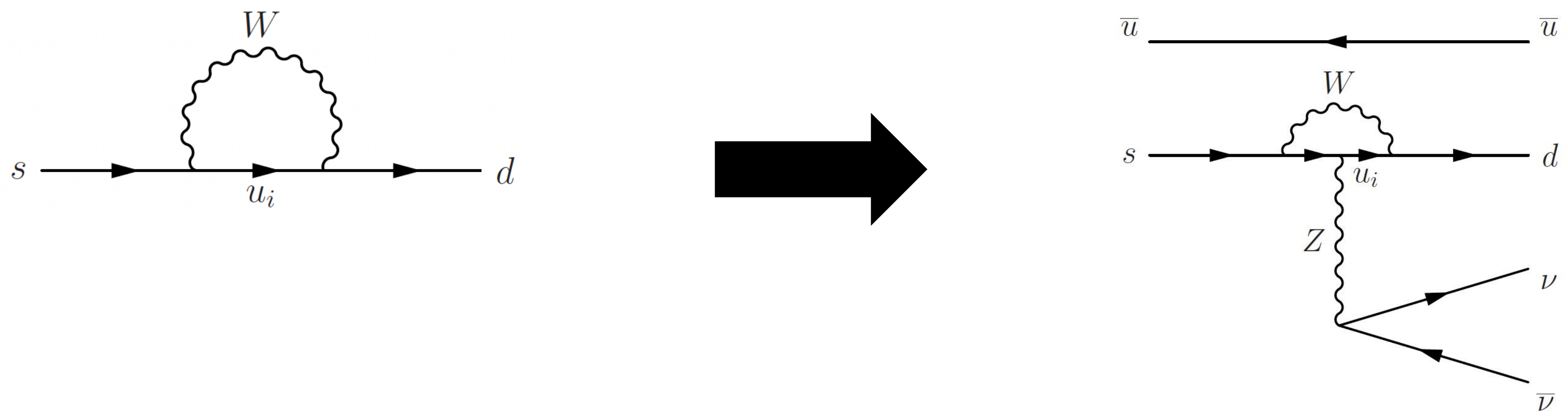




Meson mixing
($\Delta F = 2$)

CKM and GIM suppression in FCNC decays

- Let’s take as an example $s \to d$ transitions (change in flavour $\Delta s = -\Delta d = 1 \Longrightarrow \Delta F = 1$ transition)



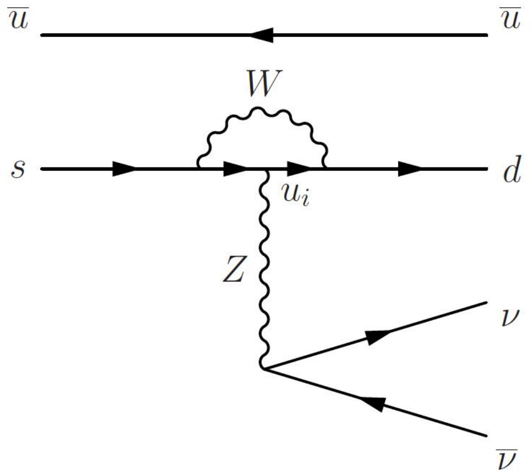
Penguin diagram contributing to $K^+ \to \pi^+ \nu \bar{\nu}$

- The flavour structure of the loop diagram is given by

$$\mathcal{A}_{s\to d} \sim \sum_{i=u,c,t} (V_{is} V_{id}^*) f(x_i)\,, \qquad x_i = \frac{m_i^2}{m_W^2}$$

depends on the specific decay

CKM and GIM suppression in FCNC decays



$$\mathcal{A}_{s\to d} \sim \sum_{i=u,c,t} (V_{is}V_{id}^*) f(x_i)\,, \qquad x_i = \frac{m_i^2}{m_W^2}$$

- We can use CKM unitarity to eliminate one of the three CKM terms in the sum

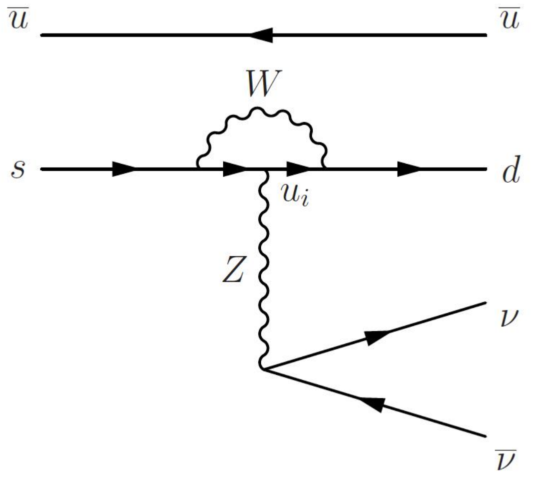
$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0$$

$$\Downarrow$$

$$\mathcal{A}_{s\to d} \sim \sum_{i=c,t} [f(x_i) - f(x_u)] V_{is} V_{id}^*$$

- **Two important lessons**
  - the contribution of the m_i-dependent terms in $f(x_i)$ to $\mathcal{A}_{s\to d}$ vanishes when summed over all internal quarks
  - $\mathcal{A}_{s\to d}$ would vanish if the up-type quarks were all degenerate and must depend on the mass-splittings among the up-type quarks

CKM and GIM suppression in FCNC decays



$$\mathcal{A}_{s\to d} \sim \sum_{i=u,c,t} (V_{is}V_{id}^*) f(x_i)\,, \qquad x_i = \frac{m_i^2}{m_W^2}$$

- In many cases for small x_i we have $f(x_i) \sim x_i$ so we can write

$$\mathcal{A}_{s\to d} \sim [(x_t - x_u)V_{ts}V_{td}^* + (x_c - x_u)V_{cs}V_{cd}^*]$$

- **Two important suppression factors**
  - *CKM suppression:* the amplitude is proportional to at least one off-diagonal CKM matrix element, which can be very small (e.g $|V_{ts}V_{td}^*| \sim \lambda^5$ and $|V_{cs}V_{cd}^*| \sim \lambda$, where $\lambda \sim 0.225$).
  - *Glashow-Iliopoulos-Maiani (GIM mechanism) suppression:* the amplitude is proportional to mass-squared differences between the up-type quarks (e.g $(x_c - x_u) \sim (m_c/m_W)^2$)

CKM and GIM suppression in FCNC decays

$\mathcal{O}(1)$ $\mathcal{O}(10^{-4})$ $\mathcal{O}(10^{-4})$ $\mathcal{O}(10^{-1})$

$$\mathcal{A}_{s\to d} \sim [(x_t - x_u)V_{ts}V_{td}^* + (x_c - x_u)V_{cs}V_{cd}^*]$$

$b \to d$: $\mathcal{O}(1)$ $\mathcal{O}(10^{-3})$ $\mathcal{O}(10^{-4})$ $\mathcal{O}(10^{-2})$

$$\mathcal{A}_{b\to q(d,s)} \sim [(x_t - x_u)V_{tb}V_{tq}^* + (x_c - x_u)V_{cb}V_{cq}^*]$$

$b \to s$: $\mathcal{O}(10^{-2})$ $\mathcal{O}(10^{-2})$

$\mathcal{O}(10^{-3})$ $\mathcal{O}(10^{-4})$ $\mathcal{O}(10^{-6})$ $\mathcal{O}(10^{-1})$

$$\mathcal{A}_{c\to u} \sim [(x_b - x_d)V_{ub}V_{cb}^* + (x_s - x_d)V_{us}V_{cs}^*]$$

$t \to u$: $\mathcal{O}(10^{-3})$ $\mathcal{O}(10^{-3})$ $\mathcal{O}(10^{-4})$ $\mathcal{O}(10^{-2})$

$$\mathcal{A}_{t\to q(u,c)} \sim [(x_b - x_d)V_{qb}V_{tb}^* + (x_s - x_d)V_{qs}V_{ts}^*]$$

$t \to c$: $\mathcal{O}(10^{-2})$ $\mathcal{O}(10^{-3})$

- The CKM suppression applies to FCNCs decay rates, but it does not necessarily apply to the corresponding branching ratios
- Branching ratios depend on the ratio between the FCNC decay rate and the full decay width which in the down sector is also CKM suppressed

CKM and GIM suppression in FCNC decays

Remarks regarding FCNCs

- $f(x_i) \sim x_i$ approximation not valid for the top quark but we can still use for purposes of demonstration

  $\left(\frac{x_t}{x_c} \sim 10^4, \text{while} \frac{f(x_t)}{f(x_c)} \approx 10^3\right)$

- The exact form of the dependence of the mass splittings is process-dependent but the amplitude always vanishes when the internal quarks are degenerate
- The size of the FCNCs amplitudes increases with the mass of the internal quark

Examples of golden rare FCNC meson decays

- **Precise SM predictions necessary for a sensitive comparison between experiment and theory**
- Rare FCNC decays with precise SM predictions are often referred to as **golden** modes

Decay type	Decay mode	Branching ratio (~)
$B \to \mu\mu$	$B_d^0 \to \mu^+\mu^-$	10^{-10}
	$B_s^0 \to \mu^+\mu^-$	3×10^{-9}
$b \to sll \;\; [l = e, \mu]$	$B^+ \to K^+ l^+ l^-$	5×10^{-7}
	$B^+ \to K^{*+} l^+ l^-$	10^{-6}
	$B_d^0 \to K_S l^+ l^-$	3×10^{-7}
	$B_d^0 \to K^{*0} l^+ l^-$	10^{-6}
$K \to \pi\nu\nu$	$K^+ \to \pi^+ \nu\bar{\nu}$	8×10^{-11}
	$K_L \to \pi^0 \nu\bar{\nu}$	3×10^{-11}

Summary of Lecture 9

Main learning outcomes

- What are the main characteristics of some of the main characteristics of tree-level semileptonic decays of mesons and how to measure their properties experimentally
- What CKM tests we can conduct using tree-level semileptonic decays of mesons and observed tensions in the data
- What are Flavour Changing Neutral Currents and how do they appear in the Standard Model